

# Thermal Use of Shallow Groundwater



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**Fritz Stauffer, Peter Bayer, Philipp Blum,  
Nelson Molina-Giraldo, and Wolfgang Kinzelbach**



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# Preface

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The thermal use of the shallow subsurface is increasingly promoted and implemented as one of many promising measures for saving energy. The energy extracted from such systems is referred to as shallow geothermal energy or low-enthalpy energy. Open and closed systems are distinguished usually consisting of boreholes combined with heat pumps. A series of questions arises with respect to the design, the management of underground and groundwater heat extraction systems, such as the sharing of the thermal resource, the long-term sustainability of the thermal use, and the assessment of its long-term potential. For the proper design of thermal systems, it is necessary to assess their impact on underground and groundwater temperatures.

The theoretical basis of heat transport in soil and groundwater systems is therefore introduced, and the essential thermal properties are discussed. In the planning and design of geothermal systems, hydrogeological and thermal site investigations have to be combined with modeling. Therefore, a series of mathematical tools and simulation models based on analytical and numerical solutions of the heat transport equation are presented. Finally, some case studies are introduced for illustration.

The book is directed toward MSc students in civil or environmental engineering, engineering geology, and hydrogeology and junior professionals. It provides a platform of principles and outlines the essential models and parameters to assess and design technical systems for the thermal use of the shallow underground.

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# Authors

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**Fritz Stauffer, Prof. Dr.**, is a retired professor at the Institute of Environmental Engineering at ETH Zurich, Switzerland. He was born in 1947 and is a citizen of Switzerland. He was a senior research officer and lecturer in groundwater and hydromechanics. He studied rural engineering at ETH Zurich, where he obtained his diploma in 1971 and his doctorate in 1977. The PhD thesis—under the supervision of Professor Dr. T. Dracos—was in the field of unsaturated flow in porous media. From 1981 to 2010, Fritz Stauffer taught courses in groundwater and organized a series of 20 international scientific short courses in groundwater management as well as several international symposia at ETH Zurich. In 1996, he was a visiting scholar at Stanford University. In 2001, he was awarded the title of professor by ETH Zurich. His research interests are in flow and contaminant transport in groundwater including the capillary zone.

Apart from using experimental techniques, he mainly focuses on mathematical modeling as well as geostatistics and stochastic modeling of flow and transport processes in highly heterogeneous aquifers. Main applications are the quantification of the uncertainty in the localization of groundwater protection zones, the interaction between rivers and aquifers, thermal processes in groundwater, the coupled flow and heat transport in porous media with phase change (as in the thawing permafrost of rock glaciers), the thermal use of groundwater, and two-phase flow processes in porous media. He retired from ETH in January 2012.

**Peter Bayer, Dr.**, is a senior research associate at the Department of Earth Sciences at ETH Zurich, Switzerland. He was born in 1972 and is a German citizen. He graduated in 1999 from the Center for Applied Geosciences of the University of Tübingen (Germany) and earned his PhD from the same institution in 2003. From 2008 to 2010, Peter Bayer was a EU Intra-European Marie Curie Fellow at ETH Zurich hosted by the Institute of Environmental Engineering. His work was aimed at the development of

algorithmic optimization procedures and their implementation to solve problems related to applied hydrogeology and geothermics.

Among his main scientific contributions in the area of thermal use of shallow aquifers are the development and application of life cycle-based concepts for analyzing environmental aspects of low-enthalpy geothermal systems. Together with coauthors P. Blum and N. Molina-Giraldo, he investigated the influence of groundwater flow on borehole heat exchangers by analytical and numerical simulation as well as through field studies. This work was accompanied by enhancements to thermal field investigation techniques in shallow aquifers, in particular, the application and interpretation of thermal response tests under conditions with significant groundwater flow velocity. Peter Bayer has been working on thermal processes in aquifers on the lab scale, the several meters field scale, as well as the large scale of urban subsurface temperature evolution. His research has been published in more than 75 scientific contributions, 50 of which are listed in the Web of Science.

**Philipp Blum, Jun.-Prof. Dr.,** born in Ulm in 1972, is currently an assistant professor (junior professor) for engineering geology at the Karlsruhe Institute of Technology (KIT). From 1993 to 1996, he studied geology at the University of Heidelberg. In 1997, as part of the Erasmus programme, he joined the School of Earth Sciences at Cardiff University in Wales. From 1996 to 2000, he continued his studies in applied geology at the University of Karlsruhe, where he received his diploma in 2000. In 2003, as part of the international research project DECOVALEX, he received his PhD on hydromechanical processes in fractured rock at the School of Earth Sciences at the University of Birmingham (UK). From 2003 to 2005, he was working for URS Germany as a project manager and hydrogeologist. From 2006 to 2010, he was an assistant professor for hydrogeothermics at the University of Tübingen (Germany), where in 2010, he received his habilitation in applied geology on thermohydromechanical and chemical (THMC) processes in porous and fractured aquifers. He published more than 40 peer-reviewed publications, of which 36 are also listed in the Web of Science. His current research interests focus on contaminant hydrogeology, shallow geothermal energy, and engineering geology in porous and fractured rocks.

**Nelson Molina-Giraldo, Dr.,** born in 1981 in Colombia, is a groundwater modeler at Matrix Solutions, Inc., Canada. He obtained his first degree in environmental engineering at the University of Antioquia, and then he moved to Germany and completed a master programme in applied environmental geosciences (AEG) at the University of Tübingen, Germany. He received his



PhD at the same university in 2011, where he conducted research into heat transport modeling in shallow aquifers. Currently, he has been working on analytical and numerical modeling to assess the feasibility of groundwater withdrawal forecasts for operational management and regulatory needs. He has been also implementing groundwater–surface water monitoring programs to attempt to measure changes in groundwater–surface water interaction based on temperature measurements.

**Wolfgang Kinzelbach, Prof. Dr.,** born in 1949 in Germany, is a full professor of hydromechanics and groundwater at ETH Zurich, Switzerland. He obtained his first degree in physics at Munich University and then turned to environmental engineering in Stanford, California. He earned his PhD at Karlsruhe University in 1978, with a thesis on managing waste heat emissions by power stations on the Rhine River. After professorships at Kassel University and Heidelberg University, he joined ETH in 1996. He has been working on groundwater themes including the modeling of pollutant transport, environmental tracers, design of remediation measures, and management of water resources for more than 20 years. His more recent work focuses on real-time modeling and control of well fields and sustainable management of water resources in arid environments. He is the author or a coauthor of more than 200 publications listed in the Web of Science. He was awarded the Henry Darcy Medal of the European Geophysical Union, the Prince Sultan International Prize for Water, and the Muelheim Water Award. He is a fellow of the American Geophysical Union.



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# Symbols

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Bold symbols represent vectors and tensors, whereas italic symbols represent scalar quantities and variables.

$-a$ :	Subscript a: air
$A$ :	Area ( $\text{m}^2$ )
$-b$ :	Subscript b: borehole
$B$ :	Boundary
$c$ :	Solute concentration in water ( $\text{kg m}^{-3}$ )
$C_m$ :	Volumetric heat capacity of porous medium or aquifer ( $\text{J m}^{-3} \text{K}^{-1}$ ) or ( $\text{W s m}^{-3} \text{K}^{-1}$ )
$c_s$ :	Specific heat capacity or specific thermal capacity, of solid material ( $\text{J kg}^{-1} \text{K}^{-1}$ )
$C_s$ :	Volumetric heat capacity of solid material ( $\text{J m}^{-3} \text{K}^{-1}$ ) or ( $\text{W s m}^{-3} \text{K}^{-1}$ )
$c_w$ :	Specific heat capacity or specific thermal capacity of water ( $\text{J kg}^{-1} \text{K}^{-1}$ ) or ( $\text{W s m}^{-3} \text{K}^{-1}$ )
$C_w$ :	Volumetric heat capacity of water ( $\text{J m}^{-3} \text{K}^{-1}$ )
$D$ :	Aquifer domain
$D_h$ :	Hydrodynamic dispersion tensor for solute transport ( $\text{m}^2 \text{s}^{-1}$ )
$D_t$ :	Thermal diffusion tensor or thermal diffusivity tensor ( $\text{m}^2 \text{s}^{-1}$ )
$D_{t,L}$ :	Longitudinal thermal diffusion coefficient ( $\text{m}^2 \text{s}^{-1}$ )
$D_{t,T}$ :	Transversal thermal diffusion coefficient ( $\text{m}^2 \text{s}^{-1}$ )
$E$ :	Energy (J) or (W s)
$-f$ :	Subscript f: fluid
$f$ :	Depth to groundwater (m)
$Fo$ :	Fourier number (–)
$g$ :	Gravitational acceleration constant (scalar) ( $\text{m s}^{-2}$ )
$\mathbf{g}$ :	Gravitational acceleration gradient ( $\text{m s}^{-2}$ )
$H$ :	Length of vertical borehole heat exchanger (m)
$h_w$ :	Piezometric head (m)
$-i$ :	Subscript i: ice
$I$ :	Recirculation rate between two wells ( $\text{m}^3 \text{s}^{-1}$ )
$I_{\text{hor}}$ :	Horizontal flow gradient (–)

$j$ :	Specific heat flux ( $\text{W m}^{-2}$ )
$J$ :	Heat flux ( $\text{W}$ ) or ( $\text{J s}^{-1}$ )
$j_{\text{disp}}$ :	Dispersive (specific) heat flux ( $\text{W m}^{-2}$ )
$k$ :	Permeability of aquifer (tensor) ( $\text{m}^2$ )
$K_w$ :	Hydraulic conductivity of aquifer (tensor) ( $\text{m s}^{-1}$ )
$L$ :	Length scale ( $\text{m}$ )
$L_f$ :	Latent heat of melting/freezing ( $\text{J kg}^{-1}$ ), $3.34 \cdot 10^5 \text{ J kg}^{-1}$ for water/ice
$m$ :	Aquifer thickness ( $\text{m}$ )
$m_{\text{VG}}$ :	van Genuchten parameter (–)
$-n$ :	Subscript n: normal direction
$\mathbf{n}$ :	Unit normal vector ( $\text{m}$ )
$N$ :	Recharge rate per unit surface area ( $\text{m s}^{-1}$ )
$n_{\text{VG}}$ :	van Genuchten parameter (–)
$-p$ :	Subscript p: pipe
$p_b$ :	Air entry pressure ( $\text{Pa}$ )
$P_t$ :	Heat production per unit volume ( $\text{W m}^{-3}$ )
$p_w$ :	Water pressure ( $\text{Pa}$ )
$\mathbf{q}$ :	Specific discharge vector, water discharge rate through unit area (gradient) ( $\text{m}^3 \text{m}^{-2} \text{s}^{-1}$ )
$Q$ :	Water discharge rate ( $\text{m}^3 \text{s}^{-1}$ )
$q_{\text{tb}}$ :	Heat flow rate per unit length of the borehole ( $= J/H$ ) ( $\text{W m}^{-1}$ )
$R$ :	Dimensionless cylindrical radius (–)
$R_c$ :	Solute retardation factor (–)
$R_{\text{t-ret}}$ :	Thermal retardation factor (–)
$R_{\text{tb}}$ :	Thermal borehole resistance ( $\text{K W}^{-1} \text{m}^{-1}$ )
$R_{\text{tw}}$ :	Thermal radius of influence ( $\text{m}$ )
$R_w$ :	Radius of influence of a well ( $\text{m}$ )
$-r$ :	Subscript r: residual
$r$ :	Radius ( $\text{m}$ )
$r_b$ :	Borehole radius ( $\text{m}$ )
$r_p$ :	Pipe radius ( $\text{m}$ )
$-s$ :	Subscript s: solid
$s$ :	Length ( $\text{m}$ )
$S$ :	Storativity of aquifer, specific yield of unconfined aquifer (–)
$S_s$ :	Specific storativity of aquifer ( $\text{m}^{-1}$ )
$S_w$ :	Saturation degree of water (–)
$S_{w,r}$ :	Residual saturation degree of water (–)
$-t$ :	Subscript t: thermal
$t$ :	Time ( $\text{s}$ )
$T$ :	Temperature ( $^{\circ}\text{C}$ or $\text{K}$ ; $0^{\circ}\text{C} = 273.15 \text{ K}$ )
$T_0$ :	Initial or undisturbed temperature ( $^{\circ}\text{C}$ )
$T_{\text{inj}}$ :	Injection temperature ( $^{\circ}\text{C}$ )
$\mathbf{u}$ :	Mean flow velocity (gradient) ( $\text{m s}^{-1}$ )
$\mathbf{u}_t$ :	Thermal velocity (gradient) ( $\text{m s}^{-1}$ )

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$_{-v}$ :	Subscript v: vapor
$V$ :	Volume ( $\text{m}^3$ )
$\nu f$ :	Volumetric fraction (–)
$_{-w}$ :	Subscript w: water
$w$ :	Water source/sink term, water volume per unit aquifer volume and unit time ( $\text{m}^3 \text{ m}^{-3} \text{ s}^{-1}$ )
$W$ :	Source/sink term in two-dimensional flow equation ( $\text{m}^3 \text{ m}^{-2} \text{ s}^{-1}$ )
$x$ :	$x$ -coordinate (m)
$\mathbf{x}$ :	Location vector (m), with coordinates ( $x, y, z$ )
$y$ :	$y$ -coordinate (m)
$z$ :	$z$ -coordinate (m), positive upward
$\alpha$ :	Angle (rad or $^\circ$ )
$\alpha_L$ :	Longitudinal dispersivity for solute transport (m)
$\alpha_T$ :	Transversal dispersivity for solute transport (m)
$\alpha_{VG}$ :	Van Genuchten parameter (m)
$\beta$ :	Angle (rad or $^\circ$ )
$\beta_L$ :	Longitudinal thermal dispersivity of aquifer (m)
$\beta_T$ :	Transversal thermal dispersivity of aquifer (m)
$\gamma$ :	Euler's constant (–) = 0.5772...
$\Delta$ :	Finite increment
$\theta$ :	Dimensionless temperature (–)
$\theta_a$ :	Volumetric air content ( $\text{m}^{-3} \text{ m}^{-3}$ )
$\theta_i$ :	Volumetric ice content ( $\text{m}^{-3} \text{ m}^{-3}$ )
$\theta_w$ :	Volumetric water content ( $\text{m}^{-3} \text{ m}^{-3}$ )
$\lambda_{BC}$ :	Pore distribution index in the model of Brooks and Corey (–)
$\lambda_{\text{decay}}$ :	First-order decay coefficient for solute transport ( $\text{s}^{-1}$ )
$\lambda_{\text{disp}}$ :	Thermal dispersion tensor ( $\text{W m}^{-1} \text{ K}^{-1}$ )
$\lambda_{\text{eff}}$ :	Effective thermal conductivity of subsurface ( $\text{W m}^{-1} \text{ K}^{-1}$ )
$\lambda_m$ :	Thermal conductivity of porous medium or aquifer ( $\text{W m}^{-1} \text{ K}^{-1}$ )
$\lambda_s$ :	Thermal conductivity of solid material ( $\text{W m}^{-1} \text{ K}^{-1}$ )
$\lambda_w$ :	Thermal conductivity of water ( $\text{W m}^{-1} \text{ K}^{-1}$ )
$\lambda_{\text{vert}}$ :	Thermal conductivity of the overburden ( $\text{W m}^{-1} \text{ K}^{-1}$ )
$\mu$ :	Dynamic viscosity (Pa s)
$\rho$ :	Dimensionless radius (m)
$\rho_a$ :	Density of air ( $\text{kg m}^{-3}$ )
$\rho_i$ :	Density of ice ( $\text{kg m}^{-3}$ )
$\rho_s$ :	Density of the solid phase of the aquifer ( $\text{kg m}^{-3}$ )
$\rho_{\text{rel}}$ :	Relative density (–)
$\rho_w$ :	Density of water ( $\text{kg m}^{-3}$ )
$\tau$ :	Period (s)
$\varphi$ :	Flow potential ( $\text{m}^2 \text{ s}^{-1}$ )
$\varphi_r$ :	Angular coordinate (polar angle) (–)
$\phi$ :	Porosity of aquifer, volumetric fraction of pores in aquifer ( $\text{m}^{-3} \text{ m}^{-3}$ or –)

$\chi$ :	Scaled pumping rate (–)
$\psi$ :	Stream function ( $\text{m}^2 \text{s}^{-1}$ )
$\omega$ :	Angular frequency ( $\text{s}^{-1}$ )
$\nabla$ :	Gradient operator, applied to scalar quantity $f$ : $\nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$
$\nabla \cdot$ :	Divergence operator, e.g., applied to vector $\mathbf{v}$ : $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$